DIPOLE STRENGTH DISPERSION
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A- PRELIMINARY

We wish to detect:

1) Trends intrinsic to magnet series (A, B, C, or D). These could be corrected by drastic design changes, ie, adjust the magnet length during stacking, or shim the gap during assembly.

2) Dispersion among magnets of the same series. These may be corrected by improving manufacturing practices in order to uniformize the relevant parameters, ie, steel quality, poletip and backleg gap, steel length tolerances, ...

B- PARAMETERS AFFECTING THE STRENGTH OF THE DIPOLES

We use an approach ad assumptions similar to the one followed by Gourber and Resegotti for the LEP dipoles.[1] If:

*magnets are always cycled up to the maximum current,
*the current is then set from zero,

the working point will be on the rising branch of the hysteresis curve:

$$B_{steel} = \mu_0 \cdot \mu_r \cdot (H + H_c) \qquad \text{Eq. 1}$$

with: μ_r : function of H only
H_c : function of Bmax only

This equation describes the steel behavior, to which we add Ampere's law, and the conservation of flux, to get, geometry apart, a full description of our magnet.

$$\frac{B_{air}}{\mu_0} \cdot \frac{gap}{2} + \int_{steel}^{o} H\, dl = N \cdot I \qquad \text{Eq. 2}$$

$$B_{air} \cdot S_{air} = B_{steel} \cdot S_{steel} \text{ (l)} \qquad \text{Eq. 3}$$

(Along a flux tube, the median plane not contributing, see picture)

(1): J. P. Gourber and L. Resegotti, IEEE Transactions on Nuc. Sci **26**, 3185

MAIN INJECTOR RING DIPOLE

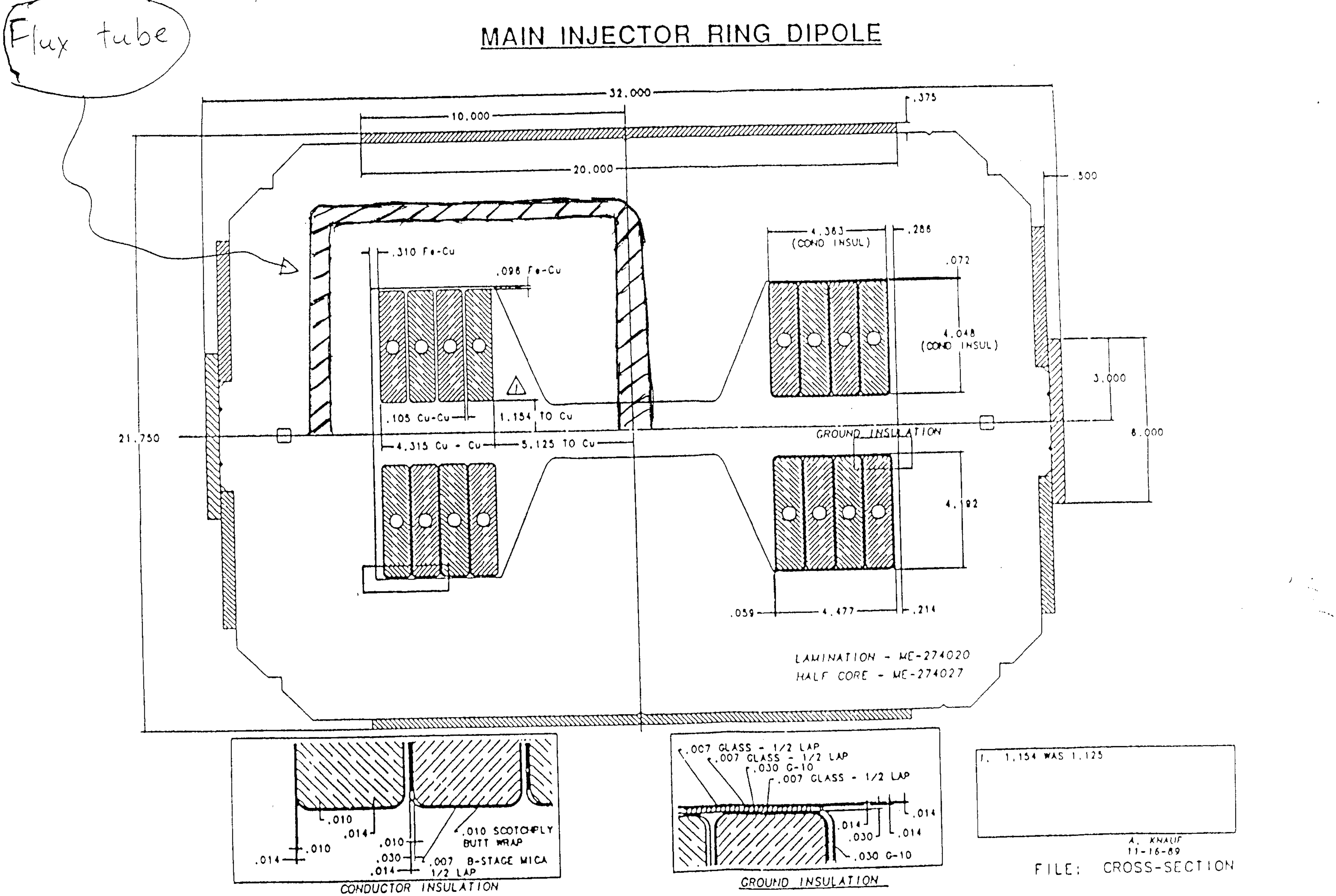

Figure 3.1-2. Cross-Section of the Main Injector Dipole Magnet

Using Eq. 1 and Eq. 2 we eliminate H and B_{steel}

$$B_{air} = \frac{2\cdot\mu_0}{gap}\cdot\left(N\cdot I + \int_{steel}^{o} H_c\, dl\right)\cdot\frac{1}{1+\frac{2}{gap}\cdot\int_{steel}^{o}\frac{S_{air}}{\mu_r\cdot S_{steel}}\, d_o} \qquad \textbf{Eq. 4}$$

The first integral can be written: $\int_{steel}^{o} H_c\, dl = L_{tube}\cdot H_c$

The second integral describes the scaling of B due to the scaling of the flux path section in both transverse and longitudinal planes. Eq. 3 can be written approximately, using the packing factor f, and three main regions in the steel, ie, the pole, the yoke, and the backleg:

$$B_{air}\cdot\frac{pole}{2} = B_{s1}\cdot\frac{pole}{2}\cdot f = B_{s2}\cdot yoke\cdot f = B_{s3}\cdot backleg\cdot f$$

(the half factor because the flux out of the gap split to the left and to the right)

$$B_{air} = \frac{2\cdot\mu_0}{gap}\cdot\left(N\cdot I + L_{tube}\cdot H_c\right)\cdot\frac{1}{1+\frac{2}{gap\cdot f}\cdot\left(\frac{L_p}{\mu_p}+\frac{L_y}{\mu_y}\cdot\frac{pole}{2\cdot yoke}+\frac{L_b}{\mu_b}\cdot\frac{pole}{2\cdot backleg}\right)}$$

Eq. 5

Now, approximately:

$pole := 2\cdot(4\cdot in)$ (Around the pole center) $\qquad L_p := 6.5\cdot in$

$yoke := 5.5\cdot in$ $\qquad L_y := 12\cdot in$

$backleg := 5.5\cdot in$ $\qquad L_b := 6.5\cdot in$

and: $N := 4$ (turns/pole)

There is no much difference between the the different widths, and the path is longer in the yoke and backleg. The last equation is simplified to:

$$B_{air} = \frac{2\cdot\mu_0}{gap}\cdot\left(4\cdot I + L_{tube}\cdot H_c\right)\cdot\frac{1}{1+\frac{2\cdot L_{tube}}{gap\cdot f\cdot\mu_r}} \qquad \textbf{Eq. 6}$$

And the total strength:

$$Strength = \frac{2\cdot\mu_0\cdot length}{gap}\cdot\left(4\cdot I + L_{tube}\cdot H_c\right)\cdot\frac{1}{1+\frac{2\cdot L_{tube}}{gap\cdot f\cdot\mu_r}} \qquad \textbf{Eq. 7}$$

and L_{tube} is around 20"-27". We will use L_{tube} of 25 inch.

This equation indicates three regimes of excitation:

1) Low excitation:

The coercive force is significant compared to the current term and its dispersion generates significant strength dispersion at injection. At I = 0 it gives the remanent field.

$$B_{remanent} = \frac{2\cdot\mu_0}{gap}\cdot\left(L_{tube}\cdot H_c\right) \qquad \textbf{Eq. 8}$$

for H_c of 1 oersted $\mu_0\cdot H_c = 1\cdot gauss$ and $B_{remanent} = 25\cdot gauss$

and the strength dispersion is : $\frac{\Delta Strength}{strength} = \frac{L_{tube}\cdot\delta H_c}{4\cdot I}$ **Eq. 9**

if: $\delta H_c := (0.10\cdot oersted)\cdot\frac{1000}{4\cdot\pi}\cdot\frac{amp}{m}$ and $I := 500\cdot amp$ (injection)

then: $\frac{\Delta Strength}{strength} = 25\cdot unit$

We should definitely mix the steel during stacking.

2) High excitation:

At high excitation, packing factor and permeability variations generate strength dispersion. The highest the current, the more sensitive we are to permeability variations.

$$\mathbf{Strength} \quad \alpha \quad \frac{1}{1+\frac{2 \cdot L_{tube}}{gap \cdot f \cdot \mu_r}} \quad \alpha \quad \frac{1}{1+\frac{25}{f \cdot \mu_r}} \qquad \textbf{Eq. 10}$$

$$\frac{\Delta Strength}{Strength} = \frac{2 \cdot L_{tube}}{gap} \cdot \delta\left(\frac{1}{f \cdot \mu_r}\right) = \frac{25}{(f \cdot \mu_r)^2} \cdot \delta(f \cdot \mu_r) \qquad \textbf{Eq. 11}$$

To get an idea of how important is this contribution, we need to know what the permeability is. We fit the field at 9500 ampere.

$B_{meas} := 1.75 \cdot tesla$ and $B_{expec} = \frac{2 \cdot \mu_0}{Gap} \cdot 4 \cdot I = 1.88 \cdot tesla$

$$\frac{B_{meas}}{B_{expec}} = \frac{1}{1+\frac{25}{\mu_r}}$$

$\mu_r := \frac{25}{\left(\frac{1.88}{1.75}\right) - 1}$ $\qquad \mu_r = 336.538$

Sensitivity to permeability: $\frac{\Delta Strength}{Strength} = (2.2 \cdot unit) \cdot \delta\mu$ **Eq. 12**

Sensitivity to packing factor: $\frac{\Delta Strength}{Strength} = (0.074 \cdot unit) \cdot \left(\frac{\delta f}{10000}\right)$ **Eq. 13**

Or 14 units of packing factor for 1 unit of strength

3) Geometrical factors

The dispersion in poletip gap, backleg gap, and magnet length, creates strength differences that are uniform with excitation.

$$\frac{\Delta \mathrm{Strength}}{\mathrm{Strength}} = \frac{\delta \mathrm{gap}}{\mathrm{gap}} \cdot (2) + \frac{\delta \mathrm{Length}}{\mathrm{Length}}$$

The factor 2 is to take into account the possibility of having the same discrepancy in the backleg region. One mil **dispersion** of the gap corresponds to 20 units in strength. 30 mils tolerance for the length are equivalent to 1.3 unit in strength for the long magnets and 1.9 unit for the short.